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DIGITAL VIDEO: Sub-Nyquist sampling of PAL colour signals

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Summary

A simple method of applying sub-Nyquist sampling techniques to digital coding of PAL colour video signals is described. This technique employs a sampling frequency of twice the colour subcarrier frequency, the sampling instants being maintained at a specified phase relative to the reference subcarrier. For System I (625-line, PAL) video signals the resulting sampling frequency is about 8·87 MHz. Comb filtering in the upper part of the video band is used both to recover a standard PAL video signal and to remove unwanted alias components from the sampled signal.

The results of subjective tests indicated that for a single coding and decoding operation, broadcast-quality colour pictures can be obtained from an 8-bit per sample p.c.m. system employing this sub-Nyquist sampling frequency; with a 6-bit d.p.c.m. system the impairment was graded 'just perceptible' on critical pictures. Further tests are required to examine the suitability of this sub-Nyquist technique for general use in a broadcasting network which might include several codecs (coding and decoding operations) in tandem.

Issued under the authority of

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1. Introduction

One method of reducing the bit-rate of digitally coded signals is to reduce the sampling frequency in the encoder. According to the Nyquist sampling theorem, the minimum sampling frequency which can be used without introducing unwanted components into the decoded analogue signal is equal to twice the highest frequency of the original analogue signal: this minimum frequency is often referred to as the Nyquist sampling frequency. However, other workers have discovered techniques which enable certain sub-Nyquist sampling frequencies to be employed in the digital coding of monochrome video signals with only a slight degradation apparent in the final picture quality.[1,2] These techniques have also been applied to the digital coding of the separate luminance and colour difference components of a colour video signal[1] but they normally give unsatisfactory results when applied directly to composite colour signals, as obtained in the PAL, SECAM or NTSC systems, which employ a colour subcarrier. If certain conditions are fulfilled, however, the technique can give good results when applied directly to the digital coding of a PAL video signal.*[3] One condition is that the sampling frequency should be equal to exactly twice the colour subcarrier frequency, i.e. about 8·87 MHz for System I (PAL, 625-line) video signals.

An outline of the principles of this system is given in Section 2, and Section 3 gives the results of subjective tests concerning the picture impairment introduced by the system. The available equipment could operate with linear pulse code modulation (p.c.m.) and differential pulse code modulation (d.p.c.m.). The system as described does not give satisfactory results with SECAM or NTSC video signals.

*From an original ideal by Dr. G.J. Phillips

2. Principles of sub-Nyquist sampling

2.1 Application to monochrome signals

The principles of sub-Nyquist sampling of video signals will be explained with reference to the p.c.m. system shown in Fig. 1. The spectra of video signals at various points in this system are shown in Fig. 2.

The normal condition, in which the sampling frequency f_s is greater than twice the highest frequency f_m in the video signal, will be considered first. For this condition if the video signal applied to the p.c.m. coder has a spectrum as shown in Fig. 2(a), the sampled signal obtained from the p.c.m. decoder has a spectrum as shown in Fig. 2(b) and hence the output of the following low-pass filter with cut-off frequency f_m has the same spectrum as the original analogue signal shown in Fig. 2(a). It is assumed that the effect of quantisation in the p.c.m. process has negligible effect on the spectrum of the decoded video signal. It is also assumed that the sampled signal consists of a train of narrow pulses. (See Appendix, Section 6.1).

For sub-Nyquist sampling, the wanted components of the sampled signal overlap the unwanted components in the frequency range $(f_s - f_m)$ to f_m as shown in Fig. 2(c) and therefore the signal obtained from the low-pass filter after the decoder contains unwanted frequency components, often called 'alias' components. The techniques employed to minimise the picture impairment resulting from this overlapping of wanted and unwanted components rely on the assumption that the spectra of most analogue monochrome video signals have maxima at integral multiples of the line-scan frequency f_L as shown by the full-line in Fig. 3(a). Advantage is taken of this property by employing a sampling frequency given by

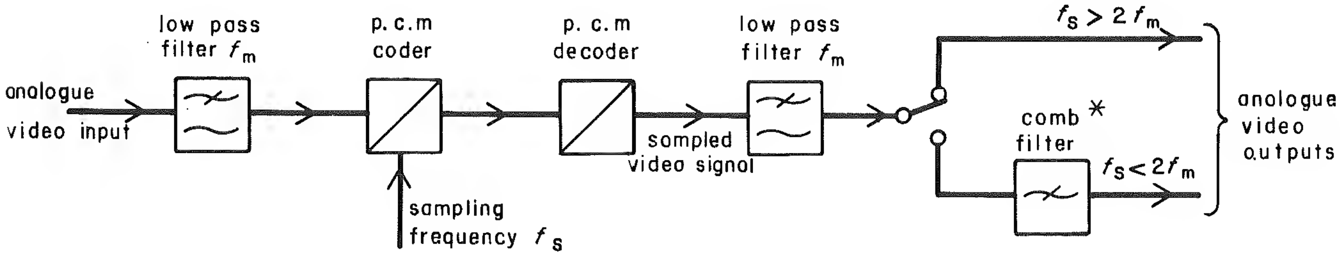


Fig. 1 - Block diagram of video p.c.m. system with sampling frequencies above and below Nyquist limit

**Comb filtering is applied to frequencies above $(f_s - f_m)$*

$$f_s = (P + ½)\ f_L \qquad (1)$$

where P is an integer. Since a wanted video component at frequency f_w causes an alias component at frequency $f_s - f_w$, the resulting spectrum of the alias components has maxima at odd harmonics of $f_L/2$ as shown by the broken line in Fig. 3(a). Under these conditions the ratio of wanted-to-alias components in the frequency range $(f_s - f_m)$ to f_m is improved by passing the sampled video signal through a comb filter with a frequency response of the form shown in Figs. 3(b). No combing is required below $(f_s - f_m)$ and therefore the overall response of the comb-filter should be as indicated in Fig. 4. (The response of a practical comb-filter is shown in Fig. 9.)

One suitable method of obtaining this frequency response is shown in Fig. 5(a). The output of this filter in the frequency range $(f_s - f_m)$ to f_m is equal to half the sum of the present input signal L_2 and the input signal delayed by one line-scan period L_1; below $(f_s - f_m)$ the output is equal to the input signal. Delay D compensates for the delay of the low-pass filter so that the subtractor has zero output for frequencies below $(f_s - f_m)$. (It is also possible to use more complex comb filters with more than one line-delay but these are not considered in this report because they may not be suitable for application to colour signals, as considered later.)

The effect on picture detail of the sub-Nyquist system described above is dependent on the angle between the picture detail and the scanning lines. Vertical transitions in the picture detail are not impaired since the resulting wanted frequency components occur at integral multiples of line frequency which are not attenuated by the comb filter and the alias components occur at the nulls in the comb filter response. With diagonal transitions

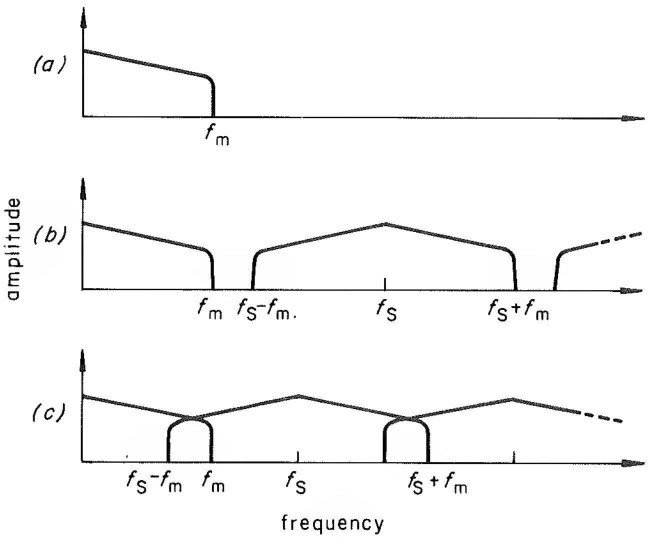


Fig. 2 - Spectra of video signal before and after sampling
(a) Spectrum of video signal before sampling
(b) Spectrum of sampled video signal for $f_s > 2f_m$
(c) Spectrum of sampled video signal for $f_s < 2f_m$

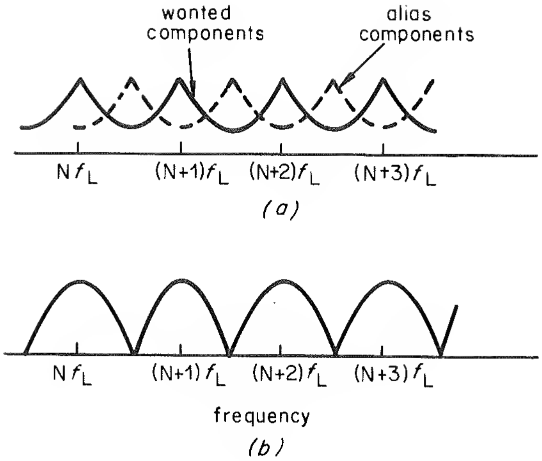


Fig. 3 - Frequency characteristics for frequency range $(f_s - f_m)$ to f_m (N is an integer).
(a) Spectra of wanted and alias components of video signal
(b) Comb filter response

however, the comb filter reduces the amplitude of wanted frequency components above $(f_s - f_m)$ and the corresponding alias components are not completely removed. Horizontal transitions are virtually unaffected since the resulting video frequencies normally lie in the range below $(f_s - f_m)$ in which no aliasing components are obtained and the comb filter has no effect.

2.2 Improved system for monochrome signals

With the sub-Nyquist system described in the previous section, any components at frequencies above $(f_s - f_m)$ which occur at <u>odd</u> harmonics of $f_L/2$ are removed at the decoder output by the comb filter but, if they are present in the original signal, they give alias components which are not attenuated since they occur at the maxima of the comb response. The ratio of wanted-to-unwanted components is therefore further improved if these components are removed prior to sampling by a comb filter with a similar frequency response to the comb filter used after sampling. The design of both comb filters could be as shown in Fig. 5(a) but the resulting system would have the disadvantage that the overall mean delay of high-

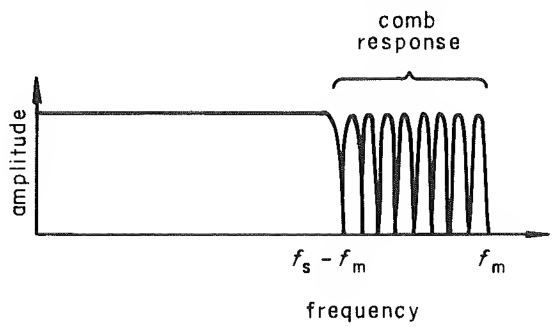


Fig. 4 - Overall response of comb filter

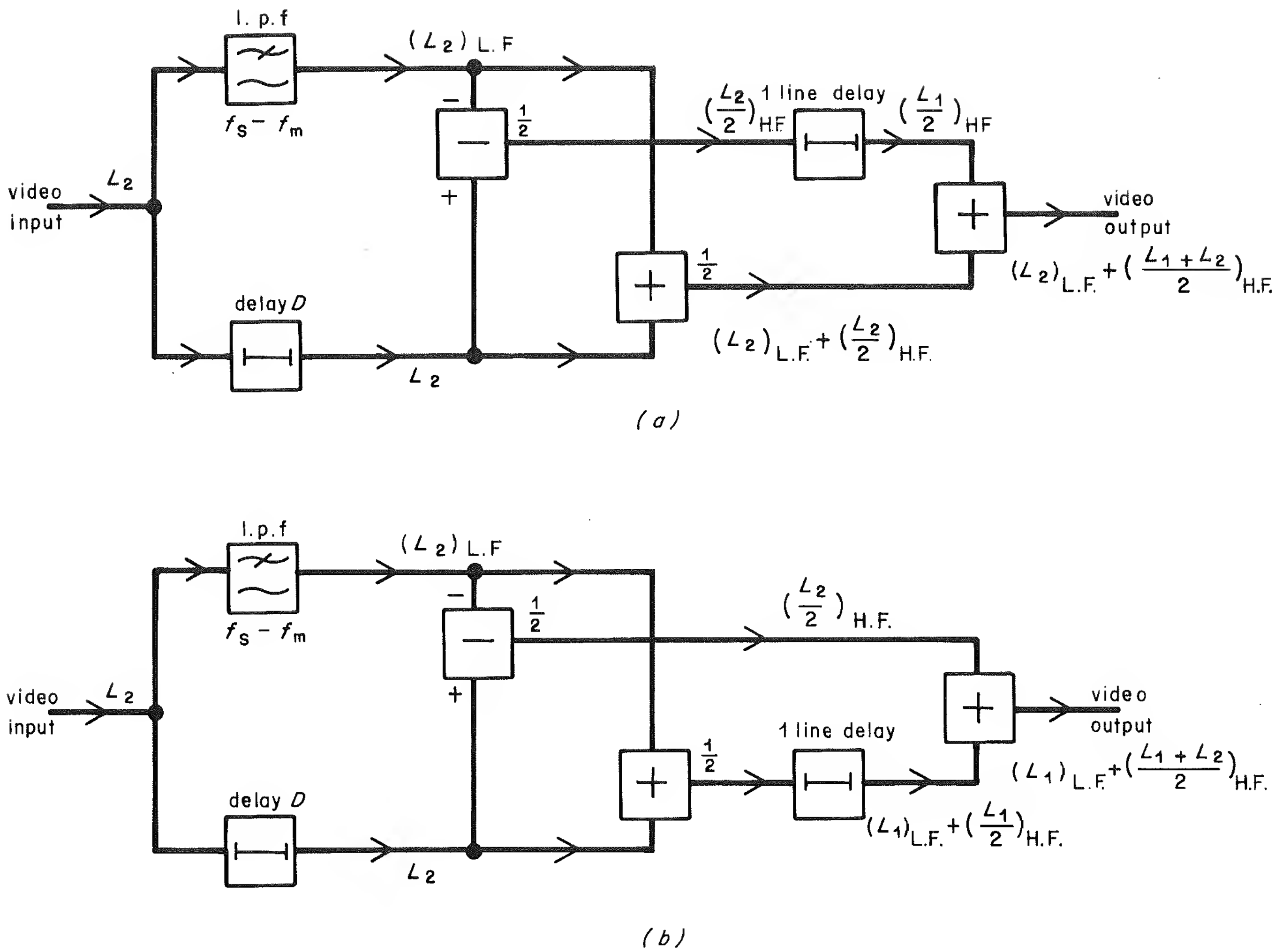


Fig. 5 - Two forms of comb-filter using a one-line delay. L_1 *and* L_2 *respectively indicate signals on two successive lines*
(a) Low frequencies undelayed *(b) Low frequencies delayed by one line-scan period*

frequency components would be one-line scan period greater than the delay at low frequencies. This problem can be overcome by using the design shown in Fig. 5 (b) before sampling while keeping the design of Fig. 5(a) for the filter after sampling, (or vice versa); the resulting system has the same delay at both low and high frequencies. The only difference in the transfer characteristics of the two filters is an extra delay of one line-scan period at low frequencies for the filter shown in Fig. 5(b).

To summarise, the main effect on picture quality of the additional comb filter before sampling is a reduction of the unwanted components appearing on sharp diagonal transitions in picture detail; this advantage is obtained at the expense of a further slight loss of diagonal resolution or, more precisely, an increase in the range of angles of diagonal transitions for which the resolution is reduced by a specified amount.

2.3 Application to PAL colour video signals

Before proceeding further, it is convenient at this point to define symbols to be used relating to a PAL video signal. The chrominance component of a PAL signal is given by the sum of two quadrature subcarrier signals to be denoted by u and v which are defined by the equations

$$u = U \sin 2\pi f_{sc} t \quad \ldots\ldots \quad (2)$$

$$v = \pm V \cos 2\pi f_{sc} t \quad \ldots\ldots \quad (3)$$

where U and V are proportional to B-Y and R-Y, Y being the luminance component of the video signal and B, R are the blue and red primary colour signals. The sign of v is alternately positive and negative during alternate line

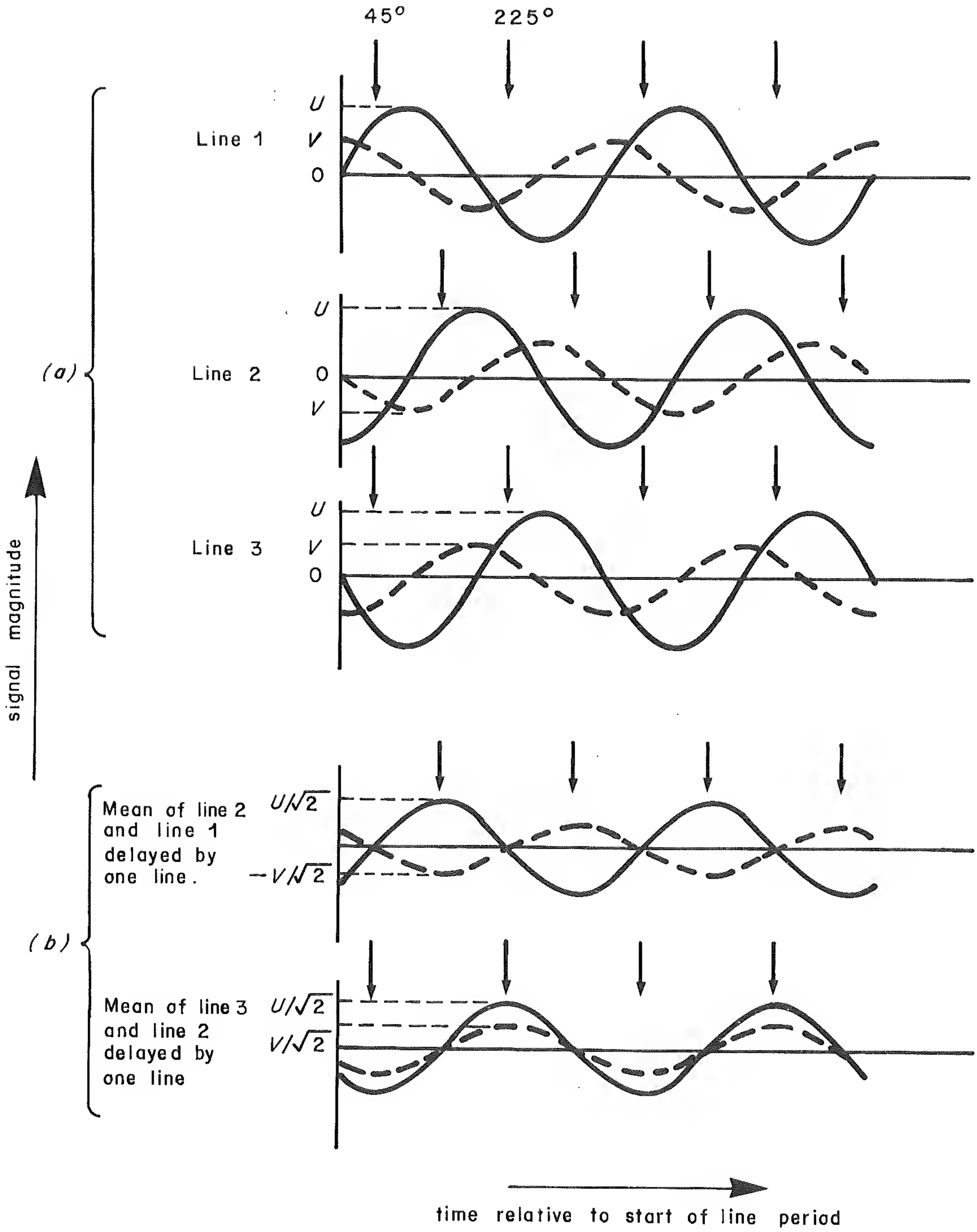


Fig. 6 - Timing of sampling relative to chrominance components of video signal. Arrows indicate sampling instants

(a) Sampling of PAL signal (b) Sampling of video signal from comb filter

——— *indicates u chrominance components* — — — *indicates v chrominance components*

periods; f_{sc} is the colour subcarrier frequency. Using this notation, the signal voltage E_m for a composite PAL signal is defined by the equation

$$E_m = Y + U \sin 2\pi f_{sc} t \pm V \cos 2\pi f_{sc} t \quad (4)$$

Sub-Nyquist sampling of composite colour signals normally gives unsatisfactory results due to difficulties arising from the presence of a colour subcarrier. With PAL colour signals, peaks in the spectrum of the modulated subcarrier usually occur very near to frequencies given by (Q ± ¼) f_L where Q is an integer. As a result, if the monochrome requirement that the sampling frequency should be of the form (P + ½) f_L is maintained, the alias components given by the chrominance signal coincide with other wanted colour components and cannot be removed by comb filter-

ing. This difficulty with PAL signals can be largely overcome by a system having the following main features.

(a) The sampling frequency must be exactly equal to twice the PAL colour subcarrier frequence.

(b) The phase of the sampling instants during a cycle of the u component of the colour subcarrier signal must be adjusted to occur at the 45° and 225° points indicated in Fig. 6(a) i.e. at times given by $2\pi f_{sc}t = \pi/4 \pm N\pi$ in Equation 4 where N is an integer.

(c) At the receiving end, the PAL signal is reconstructed from the sample values by means of a comb filter of the type shown in Fig. 5(b).* This filter should have a comb response for frequencies above $(2f_{sc} - f_m)$.

As explained in Section 2.4, improved performance is obtained by additional comb-filtering before sampling but the less complex system with only one comb filter will be discussed first.

For a p.c.m. system, a block diagram of the equipment would be as shown in Fig. 1 for $f_s < 2f_m$. For PAL signals, $2f_{sc}$ is very close to being of the form (P + ½) f_L which, as mentioned in Section 2.1, is a desirable relationship with f_L for sub-Nyquist sampling of monochrome signals (the precise value of $2f_{sc}$ for System I video signals is $567\tfrac{1}{2}f_L$ + 50 Hz). The system described above therefore has the required features for sub-Nyquist sampling of monochrome signals and is thus suitable for processing the luminance component of colour signals.

It will be shown that the resulting sampled signal from the p.c.m. decoder contains a non-standard colour signal in which the phase of the subcarrier is restricted to one axis. The comb filter following the decoder not only assists in the removal of 'alias' component caused by the luminance signal but also converts the non-standard colour signal into a standard PAL signal.

The effect of $2f_{sc}$ sampling on the chrominance components of the PAL signals may be explained with reference first to Fig. 6(a) and then to Fig. 7(a). Both these figures indicate the chrominance components of a PAL signal during successive lines 1, 2, 3 etc. in one field period. Also, both figures apply to an area of uniform colour i.e. $U_1 = U_2 = U_3 = U$ and $V_1 = V_2 = V_3 = V$ where suffixes 1, 2 and 3 refer to lines 1, 2 and 3. By examination of Fig. 6(a), it may be seen that, after sampling, the amplitude of the subcarrier is proportional to $(U + V)$ on odd lines and $(U - V)$ on even lines. (The change from $(U + V)$ to $(U - V)$ is caused by the 180° switching of the v component on alternate lines in a PAL signal). Moreover, with only two samples per cycle, only one phase of subcarrier is transmitted; this is at 45° to the phase of the original u component on all lines. It can be shown that the precise amplitudes of the subcarrier on alternate lines are $\sqrt{2}\ (U + V)$ and $\sqrt{2}\ (U - V)$. The reason for the factor of $\sqrt{2}$ may be roughly explained in the following manner. Firstly, the amplitude is reduced by a factor of $1/\sqrt{2}$ due to the phase of sampling being at 45° to the maxima of the u and v components. Secondly, the addition of alias components to wanted components causes a doubling of the amplitude of chrominance components. This doubling occurs because at a sampling frequency of $2f_{sc}$, a wanted component at frequency $(f_{sc} + f_c)$ will produce an alias component at $(f_{sc} - f_c)$. Since a normal PAL chrominance signal has a spectrum which is symmetrical about f_{sc}, i.e. it consists of pairs of components at $(f_{sc} + f_c)$, the alias components produced by the upper sideband are exactly superimposed on the wanted components in the lower sideband and vice-versa. Taking these two factors together gives an overall multiplying factor of $\sqrt{2}$. A more formal analysis, given in the Appendix, Section 6.1, derives the amplitude by taking account the phase of the alias component.

Vector diagrams of the phase and amplitude of chrominance components before and after sampling and also after comb filtering are shown in Fig. 7. The derivation of the components shown in Fig. 7(a) has been discussed above. Fig. 7(b) indicates the manner in which the comb filter reconstructs a standard form of chrominance signal from the sampled signal. During line 1, the comb filter adds one half the chrominance signal on line 2 at 90° to one half the chrominance signal on line 1. The 90° phase relationship arises because a one-line delay very nearly corresponds to an integral number of cycles of subcarrier minus one quarter of a cycle. For areas of uniform colour, the resultant subcarrier R in the output signal from the comb filter has the same magnitude and phase as the subcarrier in the signal before sampling.

The effect of the system on areas of non-uniform colour is considered in the Appendix, Section 6.2.

A conventional PAL waveform can also be obtained if the phase of sampling is adjusted to occur at 135° and 315° during a cycle of the u chrominance component. However, with these sampling phases, the phase of the reconstructed PAL subcarrier is shifted by 90° with respect to the original u chrominance component, and the phase of the 180° v-axis switching is inverted. If, however the comb filter of Fig. 5(a) were used, sampling should occur at 135° and 315° and sampling at 45° and 225° would give a 90° subcarrier phase shift and inversion of the 180° v component switching.

Regarding the effect on picture quality of the basic system described above, employing comb filtering only in the decoder, the luminance information is degraded in a similar manner to that described for sub-Nyquist sampling of monochrome signals, i.e. loss of diagonal resolution and addition of alias components on diagonal detail having high-frequency luminance components. The main forms of impairment to colour information are a slight loss of vertical resolution caused by the averaging of the chrominance signal on successive lines, and a slight increase in luminance-to-chrominance 'cross-colour' effects, including

*This circuit is preferable to the circuit shown in Fig. 5(a) since it offsets the vertical luminance/chrominance misregistration caused by delay-line PAL decoders whereas the circuit of Fig. 5(a) would double the misregistration.

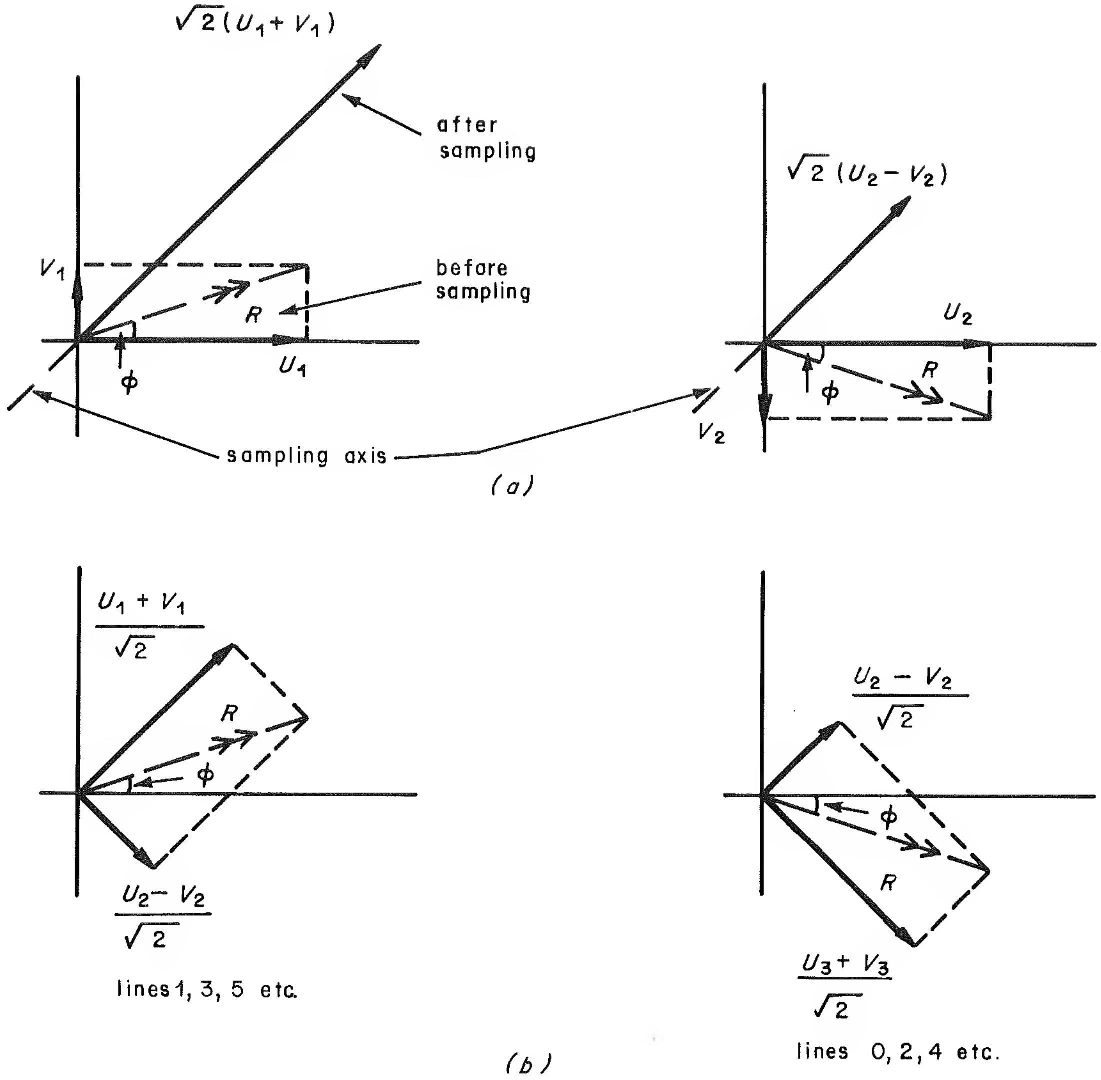


Fig. 7 - Vector diagrams showing reconstruction of PAL subcarrier after 2 f_{sc} sampling
(a) Subcarrier before and after sampling *(b) Addition of subcarrier components in comb filter*

the generation of low-frequency chrominance noise obtained by demodulation high frequency luminance noise. Also, a 12½ Hz flicker can appear on horizontal boundaries of highly saturated coloured areas (see Appendix, Section 6.2).

2.4 Improved system for PAL signals

It has been found that the use of an additional comb filter before sampling, as discussed for monochrome signals in Section 2.2, also gives improved results for PAL signals.[3]* As with monochrome signals, if the comb-filter following sampling is as shown in Fig. 5(b), the comb-filter prior to sampling should be as shown in Fig. 5(a), or vice versa. If identical filters were used before and after sampling, there would be difference on one line-period between the mean delays of low and high frequency components of the video signal which would be quite noticeable as a vertical misregistration of the colour information on horizontal coloured edges in a picture.

*From an original ideal by M. Weston.

When a PAL signal is passed through a comb filter of either type shown in Fig. 5, the phase of the chrominance component in the output signal is the same as that resulting from 2 f_{sc} sampling of a normal PAL signal, i.e. in areas of uniform colour, the phase of the subcarrier is at 45° to the u axis on all lines while its amplitude is proportional to $(U + V)$ and $(U - V)$ on alternate lines. This point is illustrated in Fig. 6(b) which shows the subcarrier waveforms in the output of a comb filter during two successive line periods. This figure indicates that in areas of uniform colour the precise amplitude of the subcarrier in the signal from the comb filter is $(U + V) / \sqrt{2}$ or $(U - V) / \sqrt{2}$. If this signal is now sampled at 2 f_{sc} so that the sampling points occur at the maxima and minima of the resulting colour subcarrier, the sample values obtained in areas of uniform colour are identical to those obtained in the system described in Section 2.3 and depicted in Fig. 7(a); therefore the same decoding procedure may be used. It is of interest to note that, with the pre-sampling comb filter, there is now only one possibility for the sampling times, namely

at 45° and 225° relative to the U component of the colour signal in the un-delayed line.

Areas of non-uniform colour are considered in the Appendix, Section 6.2.

The main improvements in the system performance conferred by the use of comb filtering before sampling are as follows.

(a) The chrominance noise and cross-colour are reduced relative to when no pre-comb filter is used and the overall system now introduces negligible impairment in these respects.
(b) Alias components on diagonal luminance edges are reduced as explained in Section 2.2.
(c) In the coder, the adjustment of the phase of sampling becomes far less critical since the comb-filtered subcarrier must be sampled at points of zero slope, i.e. at the maxima or minima.

These advantages are obtained at the expense of a further slight loss in resolution on horizontal colour boundaries and diagonal luminance transitions.

samples separated by one cycle of colour subcarrier. This time interval was selected because it is the shortest interval which can ensure small values of the difference signal in the presence of a large-amplitude colour subcarrier.

The p.c.m. coder and decoder shown in Fig. 8 included 5·5 MHz low-pass filters.

The amplitude versus frequency response of one comb filter and a 5·5 MHz low-pass filter is tandem in shown in Fig. 9. The vertical scale on this figure is linearly related to signal amplitude. Both comb filters had similar response characteristics. After processing, the video signals were displayed on a high quality colour monitor with a screen diagonal size of 560 mm (22 in.). The peak brightness on the monitor screen was adjusted to be 70 cd/m^2; with zero beam current, the brightness of the screen resulting from ambient illumination was about 0·7 cd/m^2.

All the pictures examined were obtained from a high quality 35 mm colour slide scanner. Five slides were used,

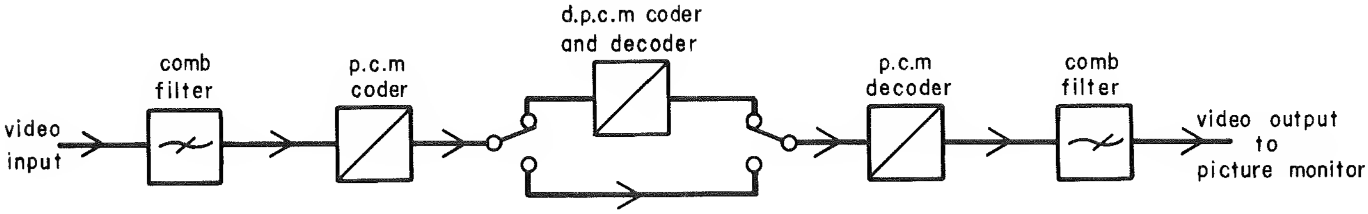


Fig. 8 - Block diagram of equipment used in subjective tests

3. Subjective tests

3.1 Equipment

Subjective tests have been performed to examine the impairment of System I PAL, 625-line, 5·5 MHz video signals resulting from both p.c.m. and d.p.c.m. coding systems employing a sampling frequency of $2f_{sc}$.

For comparison, tests were also carried out with a sampling frequency of 3 f_{sc}; since this frequency was above twice the highest video frequency, no comb filters were required. A block diagram of the equipment used in the tests at 2 f_{sc} is shown in Fig. 8. The p.c.m. code used 8 bits per sample, while the subsequent d.p.c.m. codes which were examined used 4, 5 or 6 bits per sample. (See Appendix, Section 6.3). The d.p.c.m. equipment operated entirely in the digital domain as described in a previous report,[4] and the resulting codes indicated the difference between samples separated by two or three sample periods depending on whether the sampling frequency was 2 f_{sc} or 3 f_{sc}, i.e. differences were taken between

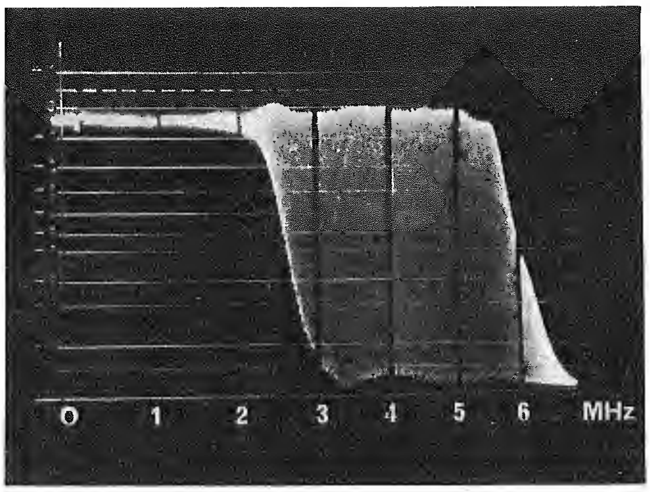
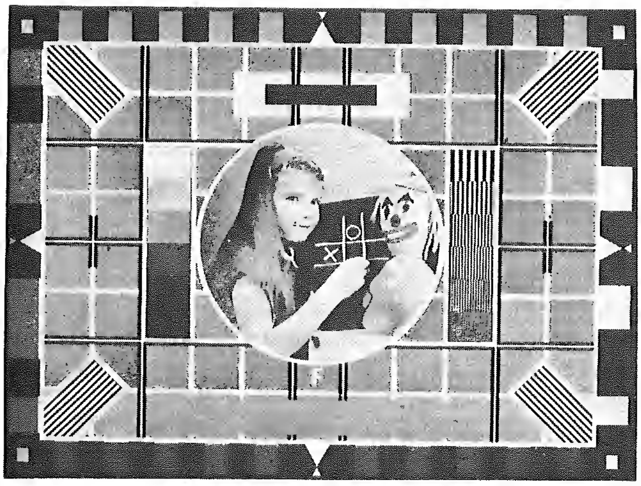


Fig. 9 - Amplitude versus frequency response of one comb filter and a 5·5 MHz low-pass filter in tandem. On a static test the attenuation at a notch at about 3·5 MHz was −45 dB

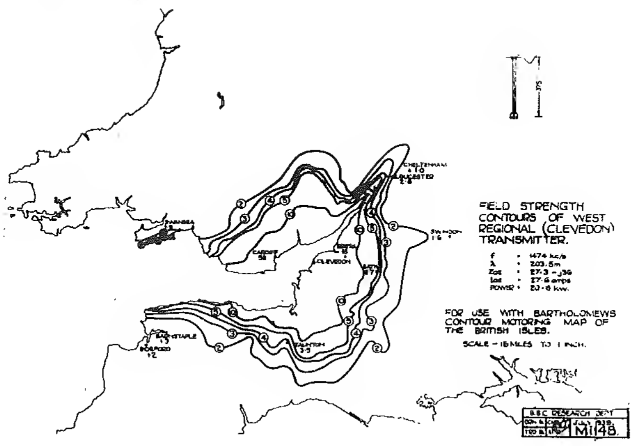


Slide 1

Slide 2

Slide 3

Slide 4



Slide 5

Fig. 10 - Slides used in subjective tests

these being selected to provide reasonable critical scenes for showing the forms of picture impairment to be expected. Monochrome versions of these slides are shown in Fig. 10.

3.2 Test procedure and results

Each test picture was shown to a group of eight observers seated at between four and six times picture height from the monitor. The observers were all engineers experienced in assessing picture quality. Before the tests commenced the type of impairment to be expected was pointed out to the observers. Each test condition consisted of the following sequence:— test picture; unprocessed analogue picture; test picture; unprocessed analogue picture. Sequences followed each other so that each test condition was presented twice at random placings in the procedure.

Picture quality was graded using the six-point impairment scale given below.*

Grade	Degree of impairment
1	Imperceptible
2	Just perceptible
3	Definitely perceptible but not disturbing
4	Somewhate objectionable
5	Definitely objectionable
6	Unusable

Details of the mean grades obtained for each test condition are given in Table 1. The figures in brackets indicate the standard deviation of the impairment grade resulting from changes in picture material. The results given in the right hand column of Table 1 are plotted in alternative forms in Figs. 11 and 12.

*The CCIR has a new recommendation (Geneva, 1974) to use a five-point scale for assessing the impairment of television pictures (CCIR Doc. 11/1009). In this scale, Grade 5 corresponds to imperceptible impairment. As a first approximation the linear relationship A5 = 5·8 — 0·8A6 can be used to transform a six-point grade A6 as used in this report into a CCIR five-point grade A5.

3.3 Discussion of results

Fig. 11 indicates that the p.c.m. and d.p.c.m. coding techniques employing 2 f_{sc} sampling impaired the picture by between 0·3 and 0·8 of a subjective grade more than the 3 f_{sc} sampling systems at a given number of bits per sample. On the other hand, if one compares in Fig. 12 the impairments at a given bit-rate, then the 2 f_{sc} sampling systems degraded pictures by an amount similar to or smaller than that given by the 3 f_{sc} sampling systems. Previous experience has shown, however, that quantising errors have a negligible effect on picture quality with an 8-bit p.c.m. system:* on this assumption, the impairment for the 2 f_{sc} sampling systems would remain at about grade 1·5 for larger numbers of bits per sample and therefore 2 f_{sc} sampling is likely to give greater impairment than 3 f_{sc} sampling at bit-rates above about 80 Mbit/s. This result assumes that similar forms of prediction have been used at both sampling rates. Improved results for d.p.c.m. with 3 f_{sc} sampling can be obtained without too much difficulty by employing two-dimensional prediction making use of samples from the previous line in the same field;[6] a similar prediction process cannot be used for 2 f_{sc} sampling since the chrominance amplitude in the sampled signal alternates between $(U + V)$ and $(U - V)$ on alternate lines. In recent tests on d.p.c.m. with 3 f_{sc} sampling and two-dimensional prediction,[7] the resulting impairment was found to be less than for the 2 f_{sc} d.p.c.m. systems using second-previous sample prediction for bit-rates above about 60 Mbit/s.

Taking grade 1·5 as the maximum acceptable impairment for broadcast-quality pictures, then the results indicate that only the 8-bit p.c.m. system was satisfactory with 2 f_{sc} sampling. It should be noted that the results given in this report apply to only one codec (combination of coder and decoder) and it has previously been considered that allowance should be made for up to four codecs in tandem in a

*The slight impairment given for 8-bit p.c.m. coding using 3 f_{sc} sampling shown in Figs. 11 and 12 and Table 1 was probably caused by factors such as the use of two sharp cut-off 5·5 MHz low-pass filters in the digital processing path rather than being the result of quantising errors.[5]

Table 1

Results of subjective tests

Sampling Frequency	Bits per sample	Code	Impairment Grade — Slide Number 1	2	3	4	5	Mean
2 f_{sc}	4	d.p.c.m.	3·21	3·79	3·79	3·86	4·00	3·73 (0·30)
	5	d.p.c.m.	2·07	2·64	2·07	2·79	2·36	2·39 (0·33)
	6	d.p.c.m.	1·93	1·79	1·71	2·07	2·36	1·97 (0·26)
	8	p.c.m.	1·36	1·57	1·36	1·64	1·64	1·51 (0·14)
3 f_{sc}	4	d.p.c.m.	2·78	2·72	3·28	2·89	2·83	2·90 (0·22)
	5	d.p.c.m.	1·89	1·83	1·89	1·50	1·89	1·80 (0·17)
	6	d.p.c.m.	1·66	1·32	1·38	1·33	1·32	1·42 (0·36)
	8	p.c.m.	1·32	1·37	1·37	1·00	1·15	1·24 (0·17)

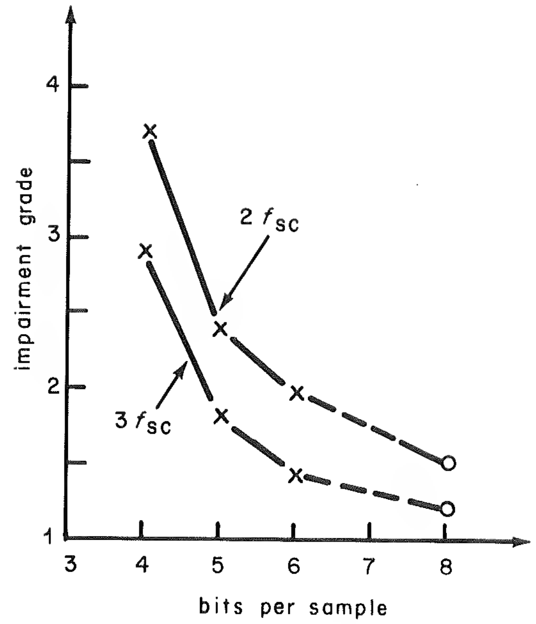


Fig. 11 - Impairment plotted against bits per sample for 2 f_{sc} and 3 f_{sc} sampling frequencies
X *indicates results for d.p.c.m.*
O *indicates results for p.c.m.*

complete broadcasting chain.[5] On the other hand, the subjective tests were performed on critical picture material and subsidiary tests showed that with most forms of broadcast pictures, it was extremely difficult to detect any impairment with the 5-bit d.p.c.m. system sampling at 2 f_{sc}.

For p.c.m., the results given in this report for bit-rate versus impairment are, to a certain extent, biased in favour of sub-Nyquist sampling since it is possible to use a lower sampling frequency than 3 f_{sc} while still remaining above the Nyquist limit. Other tests have indicated that the sampling frequency can be reduced at least to 12 MHz without causing a noticeable impairment in p.c.m. coding of 5·5 MHz PAL video signals.[8] On the other hand, with d.p.c.m. coding, the use of a sampling frequency which is not a multiple of subcarrier frequency increases the difficulty in obtaining an accurate prediction for PAL signals and hence, in general, causes an increase in quantising errors.

One limitation of sub-Nyquist sampling techniques employing comb-filters is that the averaging of signals on successive lines causes a considerable distortion of test signals which are often inserted on specific lines during the field blanking interval. However, most other bit-rate reduction techniques are unsatisfactory in this respect. One possibility of overcoming this difficulty is to employ 8-bit p.c.m. with a sampling frequency above the Nyquist limit for test signals, while bit-rate reduction techniques are applied to other video information.

4. Conclusions

This report has described a technique for applying sub-Nyquist sampling techniques to the digital coding of System I, PAL video signals. This technique employs a sampling frequency equal to exactly twice the colour subcarrier frequency (i.e. about 8·87 MHz), the phase of sampling being adjusted as described in Sections 2.3 and 2.4. Suitable comb-filtering is required to re-construct a standard PAL signal and to attenuate unwanted alias components. The process as described is not applicable to SECAM or NTSC video signals.

The results of subjective tests indicate that broadcast-quality colour pictures can be transmitted through an 8-bit p.c.m. system employing this sub-Nyquist sampling technique. Examination of critical slide pictures transmitted through a 6-bit d.p.c.m. system employing sub-Nyquist sampling revealed just perceptible picture impairments, but on most forms of broadcast pictures it was difficult to detect any impairment from a d.p.c.m. sub-Nyquist system using only 5-bits per sample. It should be noted that the results given above apply to the impairment given by only one codec (combined coding and decoding process). If up to four codecs were connected in tandem it is unlikely that the final pictures would be considered to be of broadcast quality, even with 8-bit p.c.m. in use, but this matter requires further study. For this reason, it seems improbable that the sub-Nyquist sampling process for PAL signals described in this report would be suitable for general use in digital coding of video signals throughout a broadcasting network; however, this process may find applications for purposes where the advantages of a reduced bit-rate outweigh the disadvantages of a slight picture impairment under critical viewing conditions.

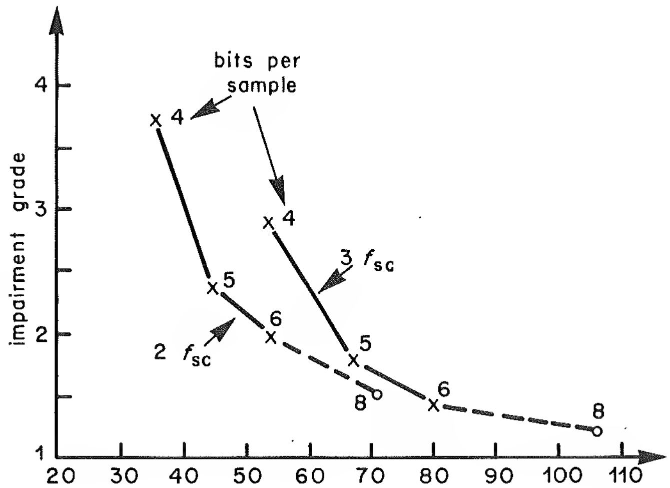


Fig. 12 - Impairment plotted against bit-rate for 2 f_{sc} and 3 f_{sc} sampling frequencies
X *indicates results for d.p.c.m.*
O *indicates results for p.c.m.*

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6. Appendix

6.1 Analysis of the chrominance signal obtained after sampling PAL video signals at twice subcarrier frequency

The effect of sampling a video signal voltage E_m will be analysed by considering an 'ideal' sampler which provides an output consisting of a train of impulses whose areas are proportional to the magnitudes of E_m at each sampling instant. By reference to Fig. 13, it can be seen that the voltage E_s corresponding to this sampled signal is given as a function of time, t, by

$$E_s(t) = K \cdot E_m(t) \cdot E_i(t) \qquad (5)$$

where E_i represents a train of constant amplitude impulses occurring at sampling instants and K is a constant.

For a composite PAL signal, E_m can be represented by the equation (see Section 2.3, Equ. (4)):–

$$E_m = Y + U \sin \omega t + V \cos \omega t \qquad (6)$$

where $\omega = 2\pi f_{sc}$

If sampling occurs at twice subcarrier frequency and one sampling pulse occurs at a time $t = \phi/\omega$, then the

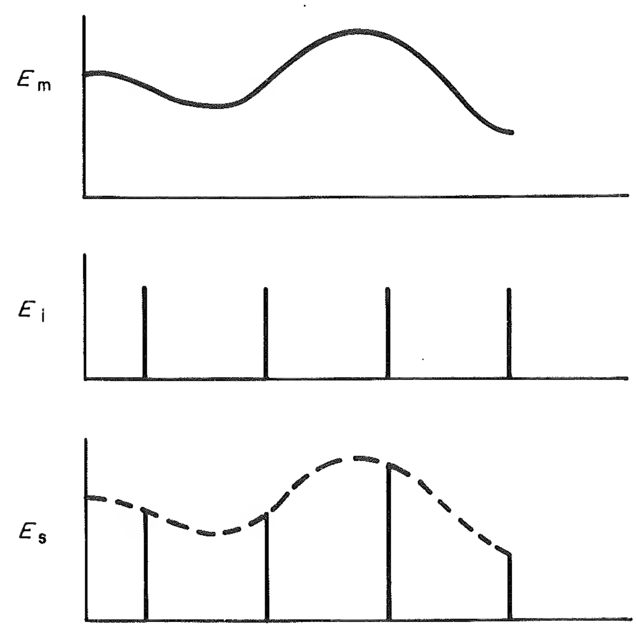

Fig. 13 - Diagram of 'ideal' sampling process

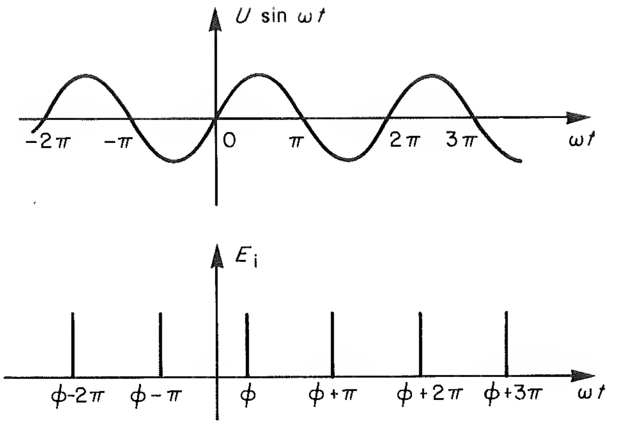

Fig. 14 - Timing of 2 f_{sc} sampling pulses relative to the u chrominance component of a PAL video signal

timing of sample pulses relative to the $U \sin \omega t$ component of E_m is as shown in Fig. 14. By Fourier analysis E_i for this pulse train is given by

$$E_i = k \left[1 + 2\cos(2\omega t - 2\phi) + 2\cos(4\omega t - 4\phi) + 2\cos(6\omega t - 6\phi) \ldots \right] \qquad (7)$$

where k is a constant.

From equations (5), (6) and (7), a PAL video signal sampled at twice subcarrier frequency can be represented by,

$$E_s = kK\,(Y + U \sin \omega t \pm V \cos \omega t) \times \left[1 + 2\cos(2\omega t - 2\phi) + 2\cos(4\omega t - 4\phi) + 2\cos(6\omega t - 6\phi) \ldots \right] \qquad (8)$$

In the twice-subcarrier sampling process described in this report, the sampled signal is passed through a low-pass filter with a cut-off frequency f_m equal to the highest frequency in the original PAL video signal. An expression for the output signal from this filter E'_s can be obtained by omitting terms in Equ. (8) corresponding to frequencies above f_m. For normal PAL system specifications, the resulting signal E'_s can only contain components given by:-

$$\frac{E'_s}{kK} = Y\left[1 + \cos(2\omega t - 2\phi)\right] + U\left[\sin \omega t - \sin(\omega t - 2\phi)\right] \pm V\left[\cos \omega t + \cos(\omega t - 2\phi)\right] \qquad (9)$$

The term $Y \cos(2\omega t - 2\phi)$ corresponds to luminance alias components and will only give frequencies below f_m if Y contains components above $(2f_{sc} - f_m)$. Since the purpose of the present analysis is to examine the effect of the sampling process on the chrominance signal, luminance alias components will be ignored by omitting $Y \cos(2\omega t - 2\phi)$. Equ. (9) can then be re-arranged to give

$$\frac{E'_s}{kK} = Y + 2(U \sin\phi \pm V\cos\phi)\sin(\omega t - \phi + \tfrac{\pi}{2}) \qquad (10)$$

If $\phi = \frac{\pi}{4}$, as indicated in Section 2.3, then

$$\frac{E'_s}{kK} = Y + \sqrt{2}\,(U \pm V)\sin(\omega t + \tfrac{\pi}{4}) \qquad (11)$$

Putting kK = 1 so that the low-frequency luminance components of E'_s have the same magnitude as those of the original PAL signal E_m (see Equ. 6), it can be seen from Equ. (11) that the amplitude of the chrominance component of E'_s is equal to $\sqrt{2}\,(U+V)$ and $\sqrt{2}\,(U-V)$ on alternate lines and that its phase lags by $\pi/4$ radians relative to the original u chrominance component on all lines. This is the result given in Section 2.3.

6.2 Chrominance signals obtained in areas of non-uniform colour

Let U_1, V_1; U_2, V_2; and U_3, V_3 represent the amplitudes of the u and v components of the colour subcarrier on successive lines 1, 2 and 3 of a PAL signal prior to digital coding.

For the sub-Nyquist system employing only one comb-filter of the type shown in Fig. 5(b) (see Section 2.3), the amplitudes of the u and v components during line 1 of the reconstructed PAL signal obtained from the comb filter are given by:-

$$\text{either } U = \frac{U_1 + U_2}{2} + \frac{V_1 - V_2}{2} \qquad (12(a))$$

$$V = \frac{V_1 + V_2}{2} + \frac{U_1 - U_2}{2} \qquad (12(b))$$

$$\text{or } U = \frac{U_1 + U_2}{2} - \frac{V_1 - V_2}{2} \qquad (13(a))$$

$$V = \underbrace{\frac{V_1 + V_2}{2}}_{\text{wanted components}} - \underbrace{\frac{U_1 - U_2}{2}}_{\text{unwanted components}} \qquad (13(b))$$

The two different forms of these values for U and V apply to the two different states of the 180° phase switching of the v component occurring on alternate lines of a PAL signal. The first terms on the right-hand side of Equations (12) and (13) represent the 'wanted' average of U and V on successive lines and the second terms represent unwanted components. It can be seen that the only difference between Equations (12) and (13) is the sign of the unwanted component.

The most noticeable picture impairment resulting from the unwanted colour components is a 12½ Hz flicker on horizontal boundaries between areas of different colour. The effect is in practice visible only at boundaries involving a highly saturated colour. The frequency of 12½ Hz results from the fact that the v-axis 180° phase switching sequence corresponding to a given line of a picture is in opposite states on successive pictures, i.e. it is repetitive at a rate of 12½ Hz. As a result the sign of the unwanted components on a given line also changes at a repetition rate of 12½ Hz. The effect on a horizontal transition between colours (u_A, v_A) and (u_B, v_B) is illustrated in Table (2): in this table the two colours are represented by the colour vectors A and B; E represents the unwanted component for which the sign reverses on successive pictures.

For sub-Nyquist system employing comb filters both before and after coding (see Section 2.4), the amplitude of the u and v components during line 2 of the reconstructed PAL signal are given by:-

$$\text{either } U = \left(\frac{U_1}{4} + \frac{U_2}{2} + \frac{U_3}{4}\right) + \left(\frac{V_1}{4} - \frac{V_3}{4}\right) \qquad (14(a))$$

$$V = \left(\frac{V_1}{4} + \frac{V_2}{2} + \frac{V_3}{4}\right) + \left(\frac{U_1}{4} - \frac{U_3}{4}\right) \qquad (14(b))$$

$$\text{or } U = \left(\frac{U_1}{4} + \frac{U_2}{2} + \frac{U_3}{4}\right) - \left(\frac{V_1}{4} - \frac{V_3}{4}\right) \qquad (15(a))$$

$$V = \underbrace{\left(\frac{V_1}{4} + \frac{V_2}{2} + \frac{V_3}{4}\right)}_{\text{wanted components}} - \underbrace{\left(\frac{U_1}{4} - \frac{U_3}{4}\right)}_{\text{unwanted components}} \qquad (15(b))$$

The remarks concerning Equations (12) and (13) are also applicable to Equation (14) and (15). The effect on a horizontal colour transition is again shown in Table 2. It will be seen that for a sharp transition, the errors produced by the system using two comb filters are half as large as those produced by the single comb filter system but they last for four lines instead of two. If the rate of change of U and V were constant over several lines, both systems would produce equal errors since in this case $(U_1 - U_3) = 2\,(U_1 - U_2)$ and $(V_1 - V_3) = 2\,(V_1 - V_2)$.

6.3 D.P.C.M. non-linear quantising laws

Details of the d.p.c.m. non-linear quantising laws used in the tests are given in Table 3. The units used for

differences in this Table have been given in terms of 8-bit p.c.m. quantum levels; the PAL video signal is divided into 256 quantum levels by the 8-bit p.c.m. coder prior to the d.p.c.m. coder.

The slight assymetry between the positive and negative halves of these laws was introduced to simplify the instrumentation required to construct a variable non-linear quantiser operating in the digital domain.

It will be noted that the six-bit law has more than 64 possible output levels. This is made possible by the use of modulo-256 arithmetic in the d.p.c.m. coding and decoding processes as described in a previous report.[4]

Table 2

Effect of 2 f_{sc} sampling of a PAL signal representing a picture containing a horizontal colour transition

Line number in picture	Colour vector before coding	Colour vector after decoding	
		System using one comb filter	System using two comb filters
N	A	A	A
N + 313	A	A	A
N + 1	A	A	A
N + 314	A	(A + B) 2 ± E	(3A + B)/4 ± E/2
N + 2	A	(A + B) 2 ± E	(3A + B)/4 ± E/2
N + 315	B	B	(A + 3B)/4 ± E/2
N + 3	B	B	(A + 3B)/4 ± E/2
N + 316	B	B	B
N + 4	B	B	B
N + 317	B	B	B

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Table 3

D.P.C.M. Quantising Laws

4-bit Law (Ref. 31/5300)		5-bit Law (Ref. 51/6431)		6-bit Law (Ref. 81/8543)	
Input Diff.	Output Diff.	Input Diff.	Output Diff.	Input Diff.	Output Diff.
64 to 255	72	96 to 255	104	32 to 255	*See below
48 to 63	55	80 to 95	87	28 to 31	29
32 to 47	39	64 to 79	71	24 to 27	25
24 to 31	27	56 to 63	59	20 to 23	21
16 to 23	19	48 to 55	51	16 to 19	17
8 to 15	11	40 to 47	43	14 or 15	14
4 to 7	5	32 to 39	35	12 or 13	12
0 to 3	1	24 to 31	27	10 or 11	10
–(1 to 4)	– 1	16 to 23	19	8 or 9	8
–(5 to 7)	– 5	12 to 15	13	7	7
–(9 to 16)	–11	8 to 11	9	6	6
–(17 to 24)	–19	6 or 7	6	5	5
–(25 to 32)	–27	4 or 5	4	4	4
–(33 to 48)	–39	2 or 3	2	3	3
–(49 to 64)	–55	1	1	2	2
–(65 to 256)	–72	0	0	1	1
		– 1	– 1	0	0
		– 2	– 2	– 1	–1
		–(3 or 4)	–3	– 2	–2
		–(5 or 6)	–5	– 3	–3
		–(7 or 8)	–7	– 4	–4
		–(9 to 12)	–9	– 5	–5
		–(13 to 16)	–13	– 6	–6
		–(17 to 24)	–19	– 7	–7
		–(25 to 37)	–27	– 8	–8
		–(33 to 40)	–35	–(9 or 10)	–9
		–(41 to 48)	–43	–(11 or 12)	–11
		–(49 to 56)	–51	–(13 or 14)	–13
		–(57 to 64)	–59	–(15 or 16)	–15
		–(65 to 80)	–71	–(17 to 20)	–17
		–(81 to 96)	–87	–(21 to 24)	–21
		–(97 to 256)	–104	–(25 to 28)	–25
				–(29 to 32)	–29
				–(33 to 256)	*See below

*For these parts of the 6-bit Law, input differences from 8N to 8N + 7 give an output difference of 8N + 3 where N = 4, ±5, ±6 etc.

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